Заключительный этап олимпиады

26.05.2020

Скороходов М.С

1	2	3	4	5	общий

① Дано:

$\vec{r} = 2t^3\vec{e_x} + 3t^2\vec{e_y} + 4t\vec{e_z}$

$t = 2$

v - ?

Решение

$\vec{v} = \frac{d\vec{r}}{dt} = 6t^2\vec{e_x} + 6t\vec{e_y} + 4\vec{e_z}$

$|\vec{v}| = \sqrt{(6t^2)^2 + (6t)^2 + 4^2}$

$v = \sqrt{24^2 + 12^2 + 4^2}$

$v = \sqrt{576+144+16} = \sqrt{4(144+36+4)} = \sqrt{16(36+9+1)}$

$= 4\sqrt{46}\ \frac{м}{с}$

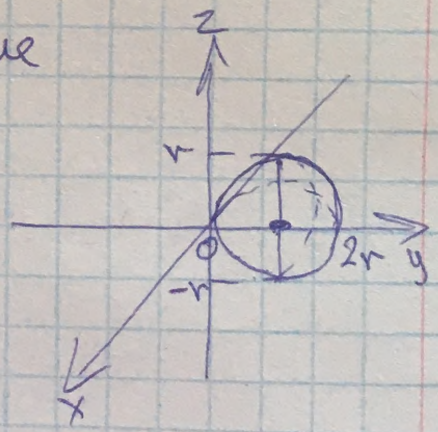
Ответ: $4\sqrt{46}\ \frac{м}{с}$

② Дано:

m, r

I - ?

Решение



$x^2 + (y+r)^2 + z^2 = r^2$

$x^2 = r^2 - (y+r)^2 - z^2$

$dI = dm\,dr^2$

$I = mR^2 \quad R^2 \to x^2$

$I = m\int_{-r}^{r}\int_{0}^{2r}(r^2 - (y+r)^2 - z^2)\,dy\,dz$

1) $\int_0^{2r}(r^2 - (y+r)^2 - z^2)\,dy = r^2y\Big|_0^{2r} - \frac{y^3}{3}\Big|_0^{2r} - 2\frac{y^2}{2}r\Big|_0^{2r} -$

$$yr^2\Big|_0^{2r} - z^2y\Big|_0^{2r} = 2r^3 - \frac{8}{3}r^3 - 4r^3 - 2r^3 - z^2\cdot 2r =$$

$$= r^3\left(2 - \frac{8}{3} - 4 - 2\right) - z^2\cdot 2r = r^3\left(-\frac{20}{3}\right) - z^2\cdot 2r$$

2) $$\int_{-r}^{r}\left[\left(-\frac{20}{3}\right)r^3 - z^2\cdot 2r\right]dz = -\frac{20}{3}r^3 z\Big|_{-r}^{r} - \frac{z^3}{3}2r\Big|_{-r}^{r} =$$

$$= -\frac{20}{3}r^3\cdot 2r - \frac{2r}{3}\cdot 2r^3 = -\frac{40}{3}r^4 - \frac{4r^4}{3} = -\frac{44}{3}r^4$$

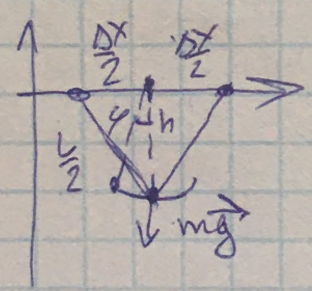
$$I = -m\cdot\frac{44}{3}r^4$$

Ответ: $-\frac{44}{3}mr^4$

③ Дано

$\Delta x, L$

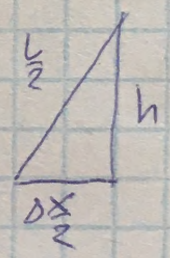
$\omega - ?$



Решение:

$M = I\ddot{\varphi}$

$I = mR^2$



$M = mgh\sin\varphi$ при $\varphi \to 0$ $\sin\varphi = \varphi$

$I = mh^2$

$mh^2\ddot{\varphi} = mgh\sin\varphi$

$\ddot{\varphi} - \frac{g}{h}\varphi = 0$ $\omega^2 = \frac{g}{h}$, $h = \sqrt{\frac{L^2}{4} - \frac{\Delta x^2}{4}}$

Ответ: $\omega = \sqrt{\frac{g}{\sqrt{\frac{L^2}{4} - \frac{\Delta x^2}{4}}}}$

4) Дано

ν

$V(T) = \alpha T^{1/3}$

V_0

$2V_0$

$A - ?$

Решение

$Q = \Delta U + A$ $\qquad$ $TdS = \Delta U + pdV$

$A = \int pdV$, т.к. $PV = \nu RT$ а $T = \frac{V^3}{\alpha^3}$

$$A = \int_{V_0}^{2V_0} \frac{\nu R V^3}{V \alpha^3} dV = \frac{\nu R}{\alpha^3} \int_{V_0}^{2V_0} V^2 dV$$

$$A = \frac{\nu R}{\alpha^3} \frac{V^3}{3} \Big|_{V_0}^{2V_0} = \frac{\nu R}{\alpha^3} \left(\frac{8V_0^3}{3} - \frac{V_0^3}{3} \right) =$$

$$= \frac{7}{3} \frac{\nu R}{\alpha^3} V_0$$

Ответ: $\frac{7}{3} \frac{\nu R}{\alpha^3} V_0$

$\frac{\Delta x^2}{4}$

5. Дано

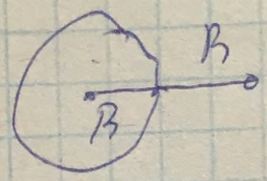
R

$\rho(r) = -\alpha r$

$L = 2R$

φ, E

Решение



$$E = \frac{1}{4\pi\varepsilon_0} = \frac{q}{r^2}$$

$$\rho = \frac{dq}{dV} \qquad dq = \rho dV$$

$$E ds = \frac{1}{\varepsilon_0} \rho ds \cdot r \quad , \quad r = 2r$$

$$E = \frac{1}{\varepsilon_0} \rho \cdot r = -\frac{\alpha}{\varepsilon_0} r^2$$

$$E = -\operatorname{grad} \varphi \qquad E = -\frac{d\varphi}{dr}$$

$$d\varphi = -E dr$$

$$\varphi = \int_0^{2r} \frac{\alpha}{\varepsilon_0} r^2 dr = \frac{\alpha}{\varepsilon_0} \frac{r^3}{3} \Big|_0^{2r}$$

$$\varphi = \frac{\alpha}{\varepsilon_0} \frac{7}{3} r^3$$

Ответ: $E = -\frac{\alpha}{\varepsilon_0} r^2$, $\varphi = \frac{7}{3} \frac{\alpha}{\varepsilon_0} r^3$.